This application is designed to illustrate the combinatorial aspects of computation of the number of orbits of a group action using the orbit-counting theorem (sometimes known as Burnside's lemma or as not Burnside's lemma). For a selected symmetry of the square, fixed colourings of a square grid are generated. The applet can be used to

- demonstrate fixed colourings of a grid for particular symmetries;
- illustrate combinatorial arguments in counting the number of fixed colourings when constraints are imposed on the colouring.

Navigation

- Select the size of grid, 3 by 3, 4 by 4 (default) or 5 by 5, using the **Size of Grid** button.
- Select a symmetry using the dropdown menu headed **e- identity element**.
- For a colouring in which a particular colour dominates, the whole grid can be coloured by selecting a colour from the palette and clicking the **Fill** button (this can be done before 2 or after 4 but not after 5).
- To colour an individual square in the grid, select a colour using the palette and click on the square.
- Click on **Apply**; the chosen square is acted upon by the selected symmetry and the resulting square is coloured in the same colour as the original. Repeat until the orbit of the chosen square is completely coloured. (Clicking **Apply** beyond this stage generates a prompting message. When the orbit of the chosen square has only one element, one application of **Apply** is required before a new square can be chosen; otherwise a new square can be chosen as soon as the final square of the orbit is coloured.)
- Choose another square and repeat.
- Continue until the grid is fully coloured.
- The **Reset** button restores the original screen.

Example To illustrate the number of colourings of a 4×4 square grid, with 12 blue squares and 4 red squares, fixed by the reflection in a diagonal:

1. Select 4×4 from the dropdown menu on the **Size of Grid** button.
2. Select reflection in the SW to NE diagonal ($y = x$) using the dropdown menu headed **e- identity element**.
3. Click on a shade of blue on the palette and click on the **Fill** button.
4. Click on a shade of red in the palette.
5. Click on any off-diagonal square, followed by **Apply**, to colour that square, and its mirror image, red. Repeat for a second off-diagonal pair. The resulting colouring is one of 15 of the first of three types of fixed colourings.

6. Choose one of the pairs and recolour it blue using the **Recent** palette and **Apply**. Select any diagonal square and colour it red. Click on **Apply** and repeat for a second diagonal square. This illustrates the second type of fixed colouring, of which there are $6 \times \binom{4}{2}$; other examples can be shown.

7. To illustrate the final type, of which there is only 1, recolour the remaining off-diagonal red squares blue and the remaining blue diagonal squares red.

8. To illustrate the full application of the orbit-counting theorem, perform the corresponding, but simpler, exercise for the other symmetries. For example, for rotation through $\frac{\pi}{2}$, steps 5,6,7 are replaced by clicking any square and colouring its orbit red by three applications of **Apply**.